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# On the collision of two shock waves in $AdS_5$

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Abstract: We consider two ultrarelativistic shock waves propagating and colliding in five-dimensional Anti-de-Sitter spacetime. By transforming to Rosen coordinates, we are able to find the form of the metric shortly after the collision. Using holographic renormalization, we calculate the energy-momentum tensor on the boundary of AdS space for early times after the collision. Via the gauge-gravity duality, this gives some insights on bulk dynamics of systems created by high energy scattering in strongly coupled gauge theories. We find that Bjorken boost-invariance is explicitely violated at early times and we obtain an estimate for the thermalization time in this simple system.

Keywords: AdS-CFT Correspondence, Classical Theories of Gravity, Hadronic Colliders.

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## Contents



## 1. Introduction

With the advent of high energy colliders with collision energies in the TeV range, progress in understanding the problem of ultrahigh energy particle scattering now involves knowledge about the dynamics of Quantum-Chromodynamics (QCD) above or close to the deconfinement scale. For instance, a new state of matter of deconfined quarks and gluons with fluid-like properties seems to have been created in Au+Au collisions at total collision energies of $\sim 40\,\mathrm{TeV}$ at the Relativistic Heavy-Ion Collider (RHIC) [1−4]. The upcoming Large Hadron Collider (LHC) will collide protons at $\sim 14\,\mathrm{TeV}$ and lead nuclei at up to $\sim 1100\,\mathrm{TeV}$. Although not a collider, the Pierre-Auger observatory probes collision energies much beyond that, at up to $\sim 10^8\,\mathrm{TeV}$.

QCD is a complicated theory to solve, especially when asking about real-time dynamics at energies close to the deconfinement scale. Traditionally one could only resort to weak coupling approaches, which due to asymptotic freedom are guaranteed to work well for bulk systems having extremely high energy densities (though not necessarily for those probed by RHIC and the LHC). In particular, phenomenological descriptions of RHIC data by applying hydrodynamic simulations with low viscosity [5] or none at all [6−8] seem to suggest that at these energy densities, the system is not weakly coupled [9, 10].

Recently, the conjectured duality between strongly coupled gauge theories and gravity [11] has opened up a new window for studying strong coupling phenomena in a range of different gauge theories (although a dual description of QCD remains elusive to date). Concerning real-time dynamics, a lot of progress has been made in calculating transport coefficients for hydrodynamics in strong coupling, such as shear viscosity [12]. However, these calculations probe the response of a static medium at finite temperature, which — while important for near-equilibrium dynamics — do not give insights into the early, far from equilibrium stages of a high energy particle collision.

From the gauge theory point of view this earliest stage following the collision has to describe the transition of the system to an equilibrium state with a well-defined temperature (if the system lives long enough), which is referred to as thermalization. The gravity dual picture of thermalization is the formation of a black hole in the bulk, its Hawking temperature being identified with the temperature in the gauge theory on the boundary of the Anti-de-Sitter (AdS) space. Thus the problem of thermalization in strongly coupled gauge theories becomes related to the problem of black hole formation, which has been noted before [13] (see also [14, 15] for a different proposal on thermalization involving black holes).

As an aside, it should be pointed out that thermalization in a high energy particle collision is not guaranteed, and even if once achieved, not easy to maintain due to the rapid (initially one-dimensional) expansion of the system.

In an inspiring work Janik and Peschanski [16] showed that an expanding thermal system corresponded to a gravity dual where the black hole was moving in the fifth dimension. Subsequent work on gauge-gravity duality in expanding systems continues to clarify the near-equilibrium late-time behavior of high energy particle collision duals [17 − 27].

Little is known about the dynamics in strongly coupled gauge theories shortly after a high energy collision, e.g. far from equilibrium. Notable exceptions are studies assuming independence from longitudinal dynamics [28], treating one space dimension instead of three [29] and a characterization of the dual horizon structure following a collision of two already deconfined plasmas [30]. We differ from [30] by using a simpler model for the incident states, which allows us to calculate the energy-momentum tensor analytically. Differences of our approach to refs. [28, 29] will be discussed below.

In this article we shall consider the problem of two colliding infinite sheets of matter in $\mathcal{N} = 4$ SYM in the large $N_c$ and large 't Hooft coupling limit. Via the gauge-gravity duality we reinterpret the problem as the collision of two shock waves on the boundary of a five-dimensional AdS space. Solving for the five dimensional metric and using holographic renormalization, one thus is able to extract information about the real time dynamics of the energy momentum tensor of the gauge theory after the collision.

Our work is organized as follows: in section 2 we construct the line-element for two colliding shock-waves in Rosen coordinates. Section 3 deals with holographic renormalization: we construct there the energy momentum tensor perturbatively in proper time and check that it is covariantly conserved and traceless. We provide a physical interpretation of our results in section 4, and put them into the perspective of the literature.

## 2. Setup and solution

Following [16], we consider for $\mathcal{N} = 4$ SYM in the strong coupling, large $N_c$ regime, an energy-momentum tensor (EMT) of the single-shock-wave-form

$$T_{++} = 0, \quad T_{+-} = 0\,, \quad T_{--} = \mu_1\, \delta(x^-)\,, \quad T_{xx} = T_{yy} = 0 \tag{2.1}$$

in light-cone coordinates $x^\pm = \frac{t \pm z}{\sqrt{2}}$. This form serves as a toy model for a large particle moving nearly at the speed of light along the $x^+$ direction with transverse energy density $\frac{dE}{d\mathbf{x}_\perp} = \mu_1$. The attribute "large" refers to the perpendicular directions, $\mathbf{x}_\perp = (x, y)$. In the gravity dual description, in Fefferman-Graham coordinates [31] this corresponds to the $AdS_5$ line element with a single shock-wave

$$ds^2 = \frac{-2dx^+dx^- + \mu_1 z^4 \delta(x^-)dx^{-2} + d\mathbf{x}_\perp{}^2 + dz^2}{z^2}\,, \tag{2.2}$$

which is an exact solution of the Einstein equations with negative cosmological constant (normalized here to $\Lambda = -6$)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 6g_{\mu\nu} = 0\,. \tag{2.3}$$

In the following we shall consider the collision of two shock-waves of the form (2.1), (2.2), corresponding to an EMT before the collision ($t < 0$) of

$$T_{++} = \mu_2\,\delta(x^+)\,, \quad T_{+-} = 0\,, \quad T_{--} = \mu_1\,\delta(x^-), \quad T_{xx} = T_{yy} = 0\,. \tag{2.4}$$

This problem was posed originally in ref. [16]. The simple question we want to address is: what is the form of the EMT after the collision, and in particular in the forward light-cone ($x^\pm > 0$)?

This setup of particle collisions in strongly coupled gauge theories mirrors closely that of Kovner, McLerran and Weigert [32] who treated collisions using classical Yang-Mills dynamics. Starting from a charge current $J^\mu = \delta^\mu_\mp \delta(x^\pm)\rho_{1,2}(\mathbf{x}_\perp)$ and solving the classical Yang-Mills equations $D_\mu F^{\mu\nu} = J^\nu$ they showed that the resulting gauge fields (and hence the EMT) were functions of the product $x^+x^-$ only, and therefore "boost-invariant" in the sense of Bjorken [33]. Quantum fluctuations are expected to break this boost-invariance [34], since even tiny fluctuations are unstable to exponential growth [35]. Our model (2.4) in some sense corresponds to the simple case $\rho_{1,2}(\mathbf{x}_\perp) = \mu_{1,2} = \text{const.}$, but is extendable to a situation where the $\rho$'s are taken from the Color Glass Condensate framework [36, 37]. We shall report on this interesting generalization in a subsequent work [38].

In order to calculate the energy momentum tensor in the forward light-cone for strongly coupled $\mathcal{N} = 4$ SYM, let us first use the coordinate transformation[1] [39]

$$x^+ = u + \frac{1}{2}\mu_1\theta(v)\;\tilde{z}^4 + 2\mu_1^2\;v\;\theta^2(v)\;\tilde{z}^6\,, \quad x^- = v\,, \quad z = \tilde{z} + 2\mu_1\;v\;\theta(v)\tilde{z}^3\,, \tag{2.5}$$

where $\theta(v)$ is the Heaviside step function, to bring eq. (2.2) into the so-called Rosen form,

$$ds^2 = \frac{-2dudv + d\mathbf{x}_\perp{}^2 + \left[1 + 6\mu_1\tilde{z}^2\;v\;\theta(v)\right]^2 d\tilde{z}^2}{\tilde{z}^2\left[1 + 2\mu_1\tilde{z}^2\;v\;\theta(v)\right]^2}\,. \tag{2.6}$$

This form is advantageous since it implies a metric that is continuous across the light-like hypersurface $v = 0$. Indeed, since one can do a similar transformation for the second shock

[1] In some of the expressions below positive powers of the $\theta$-function will appear. For positive and negative argument the square of the $\theta$-function is equivalent to the $\theta$-function, so the only complication arises if the argument vanishes. However, it turns out that all expressions of the type $\theta^n(x)$ are multiplied by $x^m$ with positive $m$, and thus this complication and the associated ambiguity of defining $\theta^n(0)$ is of no relevance here.

wave, the precollision line element can be written as a simple superposition of the two (c.f. [39]),

$$ds^2 = \frac{-2dudv + d\mathbf{x}_\perp{}^2 + \left[1 + 6\mu_1 \tilde{z}^2\ v\ \theta(v) + 6\mu_2 \tilde{z}^2\ u\ \theta(u)\right]^2 d\tilde{z}^2}{\tilde{z}^2 \left[1 + 2\mu_1 \tilde{z}^2\ v\ \theta(v) + 2\mu_2 \tilde{z}^2\ u\ \theta(u)\right]^2}\,. \tag{2.7}$$

Since the metric has to be continuous and piece-wise differentiable in these coordinates, all corrections to the above line element after collision have to be proportional to $uv\,\theta(u)\,\theta(v)$ (see appendix A for a more detailed discussion). The calculation of these corrections is most conveniently done by introducing the coordinates of proper time $\tilde{\tau} = \sqrt{2uv}$ and space-time rapidity $\tilde{\eta} = \frac{1}{2}\ln\frac{u}{v}$. In these coordinates the hyper-surface spanned by $u = v = 0$ becomes $\tilde{\tau} = 0$ and the condition $\theta(u)\theta(v) \neq 0$ translates into $\tilde{\tau}$ being real and positive. Introducing $\mu = \sqrt{2\mu_1\mu_2}$ and $Y = \frac{1}{2}\ln\frac{\mu_1}{\mu_2}$, we then make the following ansatz for the line element after the collision

$$\begin{aligned} ds^2 \;=\; & \frac{-\left[1 + K(\tilde{\tau},\tilde{\eta},\tilde{z})\right] d\tilde{\tau}^2 + \left[1 + L(\tilde{\tau},\tilde{\eta},\tilde{z})\right] \tilde{\tau}^2 d\tilde{\eta}^2 + \left[1 + H(\tilde{\tau},\tilde{\eta},\tilde{z})\right] d\mathbf{x}_\perp{}^2}{\tilde{z}^2 \left[1 + 2\tilde{z}^2 \mu\tilde{\tau}\cosh(Y - \tilde{\eta})\right]^2} \\ & + \frac{\left[1 + M(\tilde{\tau},\tilde{\eta},\tilde{z})\right] \left[1 + 6\tilde{z}^2\mu\tilde{\tau}\cosh(Y - \tilde{\eta})\right]^2 d\tilde{z}^2}{\tilde{z}^2 \left[1 + 2\tilde{z}^2\mu\tilde{\tau}\cosh(Y - \tilde{\eta})\right]^2}\,, \end{aligned} \tag{2.8}$$

where $K, L, H, M$ are functions that vanish at $\tilde{\tau} = 0$ and have to be determined by solving the Einstein equations (2.3).

Determining $K, L, H, M$ for all values of $\tilde{\tau}$ may be possible with existing numerical methods [41, 40, 38]. Finding full analytical solutions is much harder, so we limit ourselves to the restricted regime of early times $\tilde{\tau} \ll 1$. For this regime we use a power series ansatz in $\tilde{\tau}$ for the functions $K, L, H, M$ and determine the coefficients by solving the Einstein equations order by order in proper time. This is readily done with `GRTensor`.[2] One finds

$$\begin{aligned} K(\tilde{\tau},\tilde{\eta},\tilde{z}) \;=\; & c_1\mu^2\tilde{\tau}^2\tilde{z}^4 + \frac{182 + 10c_1}{3}\mu^3\tilde{\tau}^3\tilde{z}^6\cosh[Y - \tilde{\eta}] - \frac{5 + c_1}{3}\mu^2\tilde{\tau}^4\tilde{z}^2 \\ & + \frac{838 + 160c_1 + c_1^2 + 45c_2}{9}\mu^4\tilde{\tau}^4\tilde{z}^8 - \frac{154 + 2c_1}{3}\mu^4\tilde{\tau}^4\tilde{z}^8\cosh[2(Y - \tilde{\eta})] + \mathcal{O}(\tilde{\tau}^5) \\ L(\tilde{\tau},\tilde{\eta},\tilde{z}) \;=\; & \frac{-16 + c_1}{3}\mu^2\tilde{\tau}^2\tilde{z}^4 + \frac{94 + 2c_1}{3}\mu^3\tilde{\tau}^3\tilde{z}^6\cosh[Y - \tilde{\eta}] - \frac{5 + c_1}{3}\mu^2\tilde{\tau}^4\tilde{z}^2 \\ & + c_2\mu^4\tilde{\tau}^4\tilde{z}^8 - \frac{806 + 10c_1}{15}\mu^4\tilde{\tau}^4\tilde{z}^8\cosh[2(Y - \tilde{\eta})] + \mathcal{O}(\tilde{\tau}^5) \\ H(\tilde{\tau},\tilde{\eta},\tilde{z}) \;=\; & -2\mu^2\tilde{\tau}^2\tilde{z}^4 - \frac{22 + 2c_1}{3}\mu^3\tilde{\tau}^3\tilde{z}^6\cosh[Y - \tilde{\eta}] - \frac{5 + c_1}{3}\mu^2\tilde{\tau}^4\tilde{z}^2 \\ & - \frac{16 + 2c_1}{3}\mu^4\tilde{\tau}^4\tilde{z}^8 + \frac{8 - 2c_1}{3}\mu^4\tilde{\tau}^4\tilde{z}^8\cosh[2(Y - \tilde{\eta})] + \mathcal{O}(\tilde{\tau}^5) \\ M(\tilde{\tau},\tilde{\eta},\tilde{z}) \;=\; & 16\mu^2\tilde{\tau}^2\tilde{z}^4 - \frac{244 - 4c_1}{3}\mu^3\tilde{\tau}^3\tilde{z}^6\cosh[Y - \tilde{\eta}] + \frac{10 + 2c_1}{3}\mu^2\tilde{\tau}^4\tilde{z}^2 \\ & + \frac{1076 + 4c_1}{3}\mu^4\tilde{\tau}^4\tilde{z}^8 + \frac{824 - 8c_1}{3}\mu^4\tilde{\tau}^4\tilde{z}^8\cosh[2(Y - \tilde{\eta})] + \mathcal{O}(\tilde{\tau}^5)\,, \end{aligned} \tag{2.9}$$

[2] GRTensorII is a package that runs within Maple or Mathematica but distinct from packages distributed with Maple or Mathematica. It is distributed freely on the World-Wide-Web from the address: http://grtensor.org.

where $c_1, c_2$ are freely choosable integration constants. Amusingly, all terms of $\mathcal{O}(\tilde{\tau}^4\tilde{z}^2)$ in (2.9) cancel for the choice $c_1 = -5$; however, at higher orders there is no similar freedom, and e.g. the $\mathcal{O}(\tilde{\tau}^7\tilde{z}^2)$ terms cannot be canceled. It is straightforward, but a little tedious to calculate $K, L, H, M$ to arbitrary order in $\tilde{\tau}$. We have performed the calculation up to (including) $\mathcal{O}(\tilde{\tau}^{10})$, but believe little insight can be gained by spelling out this solution here.

The constants $c_1, c_2$ reflect a residual gauge freedom,

$$\mathcal{L}_\xi \, g_{\mu\nu} = \xi^\alpha \partial_\alpha g_{\mu\nu} + g_{\mu\alpha}\partial_\nu \xi^\alpha + g_{\alpha\nu}\partial_\mu \xi^\alpha \,. \tag{2.10}$$

To exhibit the property of $c_1$ as parameter of residual gauge transformations we focus here on the leading order, i.e., we consider only terms up to (including) order $\tilde{\tau}^2$ (with the exception of $g_{\tilde{\eta}\tilde{\eta}}$, which has to be considered up to order $\tilde{\tau}^4$). With the generator

$$\xi^{\tilde{\tau}} = \frac{c_1 \mu^2 \tilde{\tau}^3 \tilde{z}^4}{6}\,, \qquad \xi^\mu = 0 \quad \text{otherwise} \tag{2.11}$$

the gauge transformation acts on the metric as follows:

$$\mathcal{L}_\xi \, g_{\tilde{\tau}\tilde{\tau}} = -\frac{1}{\tilde{z}^2}\, c_1 \mu^2 \tilde{\tau}^2 \tilde{z}^4 + \mathcal{O}(\tilde{\tau}^3)\,, \qquad \mathcal{L}_\xi \, g_{\tilde{\eta}\tilde{\eta}} = \frac{\tilde{\tau}^2}{\tilde{z}^2}\, \frac{c_1 \mu^2 \tilde{\tau}^2 \tilde{z}^4}{3} + \mathcal{O}(\tilde{\tau}^5) \tag{2.12}$$

All other components of the metric are either not influenced at all, or only at order $\tilde{\tau}^3$. Comparison of (2.12) with (2.9) to order $\tilde{\tau}^2$ reveals that the terms generated by the residual gauge transformation (2.10) with generator (2.11) are precisely the $c_1$-dependent terms in (2.9). This shows clearly that the freedom to choose $c_1$ corresponds to a residual gauge freedom of the (partially) gauge fixed metric (2.8), and thus we should expect that physical quantities, like the EMT, are independent from $c_1$. We shall demonstrate this in the next section. A similar analysis applies to higher orders in $\tilde{\tau}$, but the corresponding generator of residual gauge transformations is considerably more complicated than (2.11).

## 3. Holographic renormalization

Having determined the solution to the metric for short times after the collision in the previous section, we focus now on extracting information about the gauge theory EMT in this section. Holographic renormalization [42] gives a simple prescription to obtain the EMT once the metric is in the Fefferman-Graham form,

$$ds^2 = \frac{g_{ij} dx^i dx^j}{z^2} + \frac{z^4 T_{ij} dx^i dx^j}{z^2} + \sum_{n=0}^{\infty} \frac{z^{6+2n} h_{ij}^{(n)} dx^i dx^j}{z^2}\,, \tag{3.1}$$

where $i$ collectively denote the coordinates on the AdS boundary ($z = 0$), $g_{ij}$ is the metric on the boundary (assumed to be Minkowski) and $T_{ij}$ the gauge theory EMT. We can achieve to bring eq. (2.8) with (2.9) into the form (3.1) by the coordinate transformation

$$\begin{aligned}
\tilde{\tau} &= \tau + \sum_{n=0}^{\infty} t_n(\tau,\eta) z^{4+2n} \\
\tilde{\eta} &= \eta + \sum_{n=0}^{\infty} e_n(\tau,\eta) z^{4+2n} \\
\tilde{z} &= z + \sum_{n=0}^{\infty} a_n(\tau,\eta) z^{3+2n} \,,
\end{aligned} \tag{3.2}$$

where the $z=0$ transformation in $\tilde{\tau}, \tilde{\eta}$ has been chosen such that $g_{ij}$ maintains the form

$$g_{ij} dx^i dx^j = -d\tau^2 + \tau^2 d\eta^2 + d\mathbf{x}_\perp^2 \ .$$

One finds that the matching to lowest orders requires

$$\begin{aligned}
a_0(\tau,\eta) &= -2\mu\tau \cosh[Y-\eta] - \frac{5+c_1}{6}\mu^2\tau^4 - \mu^3\tau^7 \cosh[Y-\eta] + \mathcal{O}(\tau^8) \\
t_0(\tau,\eta) &= \frac{1}{4}\partial_\tau a_0(\tau,\eta) \\
e_0(\tau,\eta) &= -\frac{1}{4\tau^2}\partial_\eta a_0(\tau,\eta)
\end{aligned} \tag{3.3}$$

and in turn leads to the EMT

$$\begin{aligned}
T_{\tau\tau} &= \mu^2\tau^2 - 3\mu^3\tau^5 \cosh[Y-\eta] + \frac{1}{24}\mu^4\tau^8 \left(107 + 90\cosh[2(Y-\eta)]\right) + \mathcal{O}(\tau^{10}) \\
T_{\eta\tau} &= -3\mu^3\tau^6 \sinh[Y-\eta] + \frac{45}{4}\mu^4\tau^9 \sinh[2(Y-\eta)] + \mathcal{O}(\tau^{10}) \\
\tau^{-2} T_{\eta\eta} &= -3\mu^2\tau^2 + 21\mu^3\tau^5 \cosh[Y-\eta] - \frac{3}{8}\mu^4\tau^8 \left(107 + 150\cosh[2(Y-\eta)]\right) + \mathcal{O}(\tau^{10}) \\
T_{xx} = T_{yy} &= 2\mu^2\tau^2 - 12\mu^3\tau^5 \cosh[Y-\eta] + \frac{5}{24}\mu^4\tau^8 \left(107 + 144\cosh[2(Y-\eta)]\right) + \mathcal{O}(\tau^{10}) \ .
\end{aligned} \tag{3.4}$$

Note that the gauge theory EMT is independent from the residual gauge parameters $c_1, c_2, \ldots$. Moreover, it obeys $T^\mu_\mu = 0$ and $\nabla_\mu T^{\mu\nu} = 0$, as it should, where we recall that $\nabla_\mu$ is the covariant derivative with respect to the metric $g_{ij}$.

## 4. Physics interpretation

Our main result, eq. (3.4), gives the energy momentum tensor for short times after the collision of two sheets of matter in a strongly coupled gauge theory. A few remarks are in order: a non-vanishing off-diagonal element of $T_{i\tau}$ in $\tau, \eta$ coordinates means there is a flow of energy in the direction $i$, so the EMT is not in its local rest-frame. Locally, the EMT may always be brought into its rest-frame by a Lorentz boost (e.g. $\eta \to \eta + \phi$). However, the form of eq. (3.4) is such that the EMT may not be brought into its rest-frame globally (i.e. for all $\eta$ simultaneously), since it is explicitly dependent on $\eta$. Put



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differently, the EMT (3.4) is not boost-invariant in the sense of Bjorken. This is a major difference to the result found when treating the gauge interaction as classical Yang-Mills fields (see discussion in the introduction). It may still be possible that boost-invariance is recovered (at least approximately) at late times, where our solution breaks down. The full solution obtained in a lower dimensional model [29], where boost-invariance (violated by construction at early times) is restored at late times, seems to suggest that this is the case.

Nevertheless, it is interesting to discuss the time behavior of the EMT for one particular rest-frame, e.g. "central rapidity" $\eta = Y$. Then the individual diagonal components of the EMT have the interpretation of (local) energy density ($T_{\tau\tau}$), effective longitudinal pressure ($\tau^{-2}T_{\eta\eta}$) and effective transverse pressure ($T_{xx} = T_{yy}$). Keeping in mind that the result (3.4) is valid only for $\tau > 0$ since it does not contain the original discontinuities (2.4), it is interesting to note that all components of $T_{\mu\nu}$ are very small initially and grow only proportional to $\tau^2$ (although with negative sign in the case of the longitudinal pressure). We believe that this may be due to the simplicity of our model, and more specifically to the absence of transverse ($\mathbf{x}_\perp$) dynamics in our ansatz for $\mu$ in (2.4). Along the same lines, ref. [29] found that $T_{\mu\nu}$ would vanish for all times had one started with (2.4) and completely eliminated the transverse dimensions.

At first glance, our result (3.4) contradicts that of ref. [28], where it was found that the energy density should behave as a constant for $0 < \tau \ll 1$. However, ref. [28] based their analysis on the assumption of boost-invariance, which is violated in our case, and we have neglected transverse dynamics which could change the behavior of $\lim_{\tau\to 0} T_{\tau\tau}$. It may thus be possible to reconcile our results with that of ref. [28]. On the other hand, ref. [28] forbid solutions of rising energy density by invoking the positive energy condition $T_{\mu\nu}\omega^\mu\omega^\nu \geq 0$, where $\omega^\mu$ is a time-like vector (see also [16]). While this criterion is certainly valid in classical gravity, it is somewhat questionable why it should apply to the EMT of the boundary quantum field theory. After all, it is well-known that quantum fields cannot always and everywhere satisfy all energy conditions [43], and even the weakest form of energy conditions, the averaged null energy condition, can be violated for conformally coupled quantum fields in 3+1 dimensions in any conformal quantum state [44]. It may be confusing to find quantum effects in a classical gravity calculation: however, it should be pointed out that these effects appear only at the boundary of $AdS_5$, where the description should be dual to a strongly coupled quantum field theory [11]. Indeed, the five-dimensional EMT in our calculation trivially fulfills the positive energy condition, as it should for a classical gravity calculation.

The breaking of boost-invariance of the EMT eq. (3.4) is a direct consequence of the ansatz (2.4) for the incoming shockwaves. Namely, the relevant part of the precollision line element is

$$ds^2 \sim z^2 \left[\mu_1\delta(x^-)dx^{-2} + \mu_2\delta(x^+)dx^{+2}\right], \tag{4.1}$$

which — in terms of $\tau, \eta$ coordinates — has an explicit $\eta$ dependence. It is somewhat surprising, though, that one can obtain a boost-invariant line element when formally replacing $\delta(x) \to \partial_x\delta(x)$, or similar structures.[3] In such a case, we could not find a coordinate

[3]Ref. [29] proposed $\theta(x)/x^2$ to obtain boost-invariance.

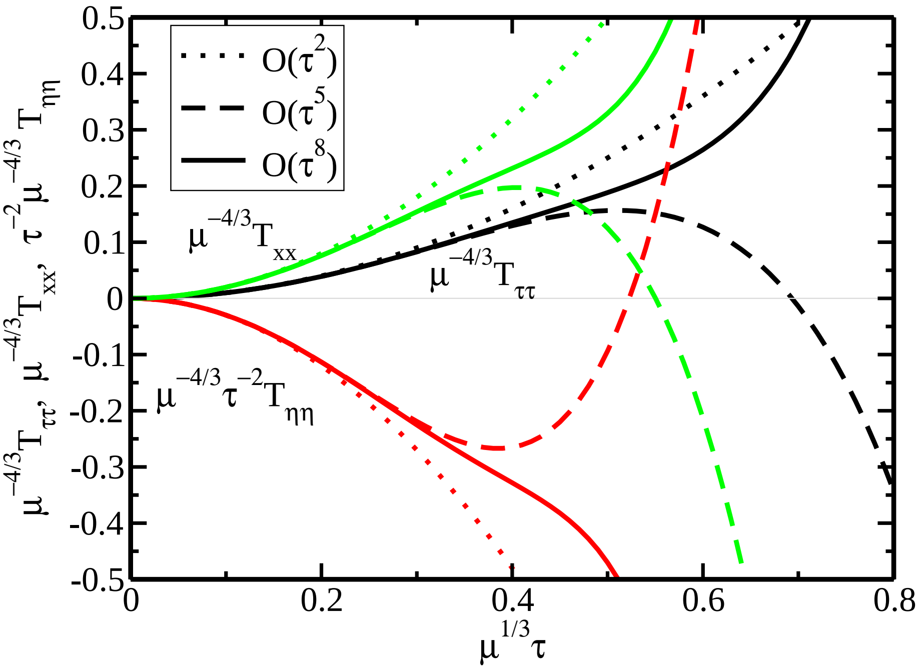


**Figure 1:** The early time behavior of the individual components of the EMT at $\eta = Y$.

transformation similar to eq. (2.5) in order to bring the line element into the Rosen form. It is conceivable that it would amount to replacing $v\theta(v)$ and $u\theta(u)$ in eq. (2.7) by their derivatives, in which case the line element would be manifestly boost-invariant and the EMT start from a finite value. However, it is not evident how to interpret the physical meaning of a collision of two shock waves given by derivatives of delta functions.

In figure 1 we show a plot of the energy density and effective pressures in the local rest-frame for early times, which suggest that our small time expansion converges rapidly up to times of $\mu^{1/3}\tau \sim 0.4$. At this time, the effective longitudinal pressure is still negative and therefore the system is clearly not in equilibrium. At times $\mu^{1/3}\tau \gtrsim 0.7$ the expansion in powers of $\tau$ seems to break down, possibly signaling the onset of a transition to an equilibrated state, which from hydrodynamics is known to behave as $T_{\tau\tau} \sim \tau^{-4/3}$.

It would be interesting to study in detail the horizon structure of eq. (2.8). Short of doing this, we may hope to learn something about the position of the horizon by invoking cosmic censorship, e.g. requiring that all singularities of the metric (2.8) are hidden behind horizons. To this end, it is instructive to consider the minimum value of $\tilde{z}$ where $H(\tilde{\tau}, \tilde{\eta}, \tilde{z}) = -1$ at early times. Approximating $H$ by its first term, we expect a singularity to appear at $\tilde{z}^4 \leq \frac{1}{2\mu^2\tau^2}$, which would have to be hidden by a horizon in order not to violate cosmic censorship. Note that this implies the horizon is moving towards the boundary at small times, in contrast to the situation at late times studied in ref. [16]. Given that in our case the energy density is initially rising and that for static systems the temperature is inversely proportional to the distance of the black brane to the boundary, this is not too surprising. As a caveat, it should be pointed out that $\tilde{z}^4 \sim \frac{1}{\mu^2\tau^2}$ is to be understood only as a very rough estimate of the singularity location since for these values of $\tilde{z}$ essentially all terms of eq. (2.9) become of the same order, even at small times.

Assuming that a singularity really does appear at $\tilde{z}^4 \sim \frac{1}{\mu^2\tau^2}$, how long would it take until the EMT at the boundary $z = 0$ "knows" about the formation of the black hole (i.e. the thermalization of the system)? We again estimate the time by assuming the information gets transported over the distance $\tilde{z}$ with the speed of light, so $\tilde{\tau} \sim \tilde{z}$. Hence we obtain a crude estimate of the thermalization time as a function of the transverse energy density $\mu$,

$$\tau_{\text{therm}} \sim \mu^{-1/3}\,, \tag{4.2}$$

where the non-trivial dimensionless prefactor is $\mathcal{O}(1)$ in our simple estimate (from figure 1, one can extract $\tau_{\text{therm}} > 0.4\mu^{-1/3}$). Taken at face value, this would imply extremely small times $\tau_{\text{therm}} \ll 1\,\text{fm/c}$ for modern colliders like RHIC or the LHC. However, it seems that numerical simulations will be necessary to provide a detailed study of thermalization and extract the dimensionless prefactor in (4.2) reliably. Moreover, in a more realistic model than the one considered here, we expect the dynamics in the transverse coordinates $\mathbf{x}_\perp$ to modify this thermalization time. We plan to study this in the near future [38].

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## A. Distributions and energy conservation

In order to address distributional issues we make the following *global* Ansatz for the line element in Rosen coordinates

$$ds^2 = \frac{-2dudvg_1(u,v,\tilde{z}) + \left(u^2dv^2 + v^2du^2\right)g_2(u,v,\tilde{z}) + g_3(u,v,\tilde{z})d\mathbf{x}_\perp{}^2 + g_4(u,v,\tilde{z})d\tilde{z}^2}{\tilde{z}^2}\,. \tag{A.1}$$

For the precollision line element we can write

$$
\begin{aligned}
g_1(u,v,\tilde{z}) &= g_3(u,v,\tilde{z}) = f_1(u,\tilde{z}) f_1(v,\tilde{z}), \\
g_2(u,v,\tilde{z}) &= 0, \\
g_4(u,v,\tilde{z}) &= f_2(u,\tilde{z}) f_2(v,\tilde{z}), \\
f_1(u,\tilde{z}) &= \left[1+2\bar{\mu}\tilde{z}^2\ u\ \theta(u)\right]^{-2}, \\
f_2(u,\tilde{z}) &= \left[1+6\bar{\mu}\tilde{z}^2\ u\ \theta(u)\right]^{2}\left[1+2\bar{\mu}\tilde{z}^2\ u\ \theta(u)\right]^{-2},
\end{aligned}
$$

where we have set $\mu_1 = \mu_2 = \bar{\mu}$ for simplicity. For this line element, the Einstein equations (2.3) do not contain any terms of the form $\delta(u), \delta(v)$ which would be singular at $u = 0, v = 0$. Nevertheless, when making an ansatz for the line element for $u > 0, v > 0$ one might be worried that this would introduce “spurious” singularities. To test for this, we choose the ansatz

$$
\begin{aligned}
g_1(u,v,\tilde{z}) &= f_1(u,\tilde{z}) f_1(v,\tilde{z}) + \mathcal{O}(uv) \\
g_2(u,v,\tilde{z}) &= \theta(u)\theta(v)\left(f_5(u,\tilde{z}) + f_5(v,\tilde{z})\right) + \mathcal{O}(uv) \\
g_3(u,v,\tilde{z}) &= f_1(u,\tilde{z}) f_1(v,\tilde{z}) + uv\theta(u)\theta(v)\left(f_6(u,\tilde{z}) + f_6(v,\tilde{z})\right) + \mathcal{O}(u^2v^2) \\
g_4(u,v,\tilde{z}) &= f_2(u,\tilde{z}) f_2(v,\tilde{z}) + uv\theta(u)\theta(v)\left(f_7(u,\tilde{z}) + f_7(v,\tilde{z})\right) + \mathcal{O}(u^2v^2),
\end{aligned}
\tag{A.2}
$$

where $f_5(u,\tilde{z}), f_6(u,\tilde{z}), f_7(u,\tilde{z})$ are required to be non-singular at $u = 0$, and we neglected terms of higher order that result in regular terms in (2.3). Requiring that all terms of the form $\delta(u), \delta(v)$ cancel in the Einstein equations gives the condition

$$
\begin{aligned}
&\left(f_5(v,\tilde{z}) + f_5(0,\tilde{z})\right) 24\bar{\mu}^2\tilde{z}^4 v^2\theta(v)^2 \frac{(1+6\bar{\mu}\tilde{z}^2 v\theta(v))}{(1+2\bar{\mu}\tilde{z}^2 v\theta(v))} - f_7(v,\tilde{z}) - f_7(0,\tilde{z}) \\
&\qquad -2\left[f_6(v,\tilde{z}) + f_6(0,\tilde{z})\right](1+6\bar{\mu}\tilde{z}^2 v\theta(v))^2 = 0,
\end{aligned}
\tag{A.3}
$$

and likewise for $v \leftrightarrow u$. If eq. (A.3) is fulfilled, then the equations (2.3) are regular at $u = 0, v = 0$ and can be conveniently solved in $\tau, \eta$- coordinates. By recasting our solution (2.8) into the form (A.2), we find

$$
\begin{aligned}
f_5(v,\tilde{z}) + f_5(0,\tilde{z}) &= \frac{1}{(1+2\bar{\mu}\tilde{z}^2 v\theta(v))^2} \lim_{u\to 0} \frac{L-K}{2uv} \\
f_6(v,\tilde{z}) + f_6(0,\tilde{z}) &= \frac{1}{(1+2\bar{\mu}\tilde{z}^2 v\theta(v))^2} \lim_{u\to 0} \left[\frac{8z^4\bar{\mu}^2}{(1+2\bar{\mu}\tilde{z}^2 v\theta(v))} + \frac{H}{uv}\right] \\
f_7(v,\tilde{z}) + f_7(0,\tilde{z}) &= \frac{(1+6\bar{\mu}\tilde{z}^2 v\theta(v))^2}{(1+2\bar{\mu}\tilde{z}^2 v\theta(v))^2} \lim_{u\to 0} \left[\frac{8z^4\bar{\mu}^2}{(1+2\bar{\mu}\tilde{z}^2 v\theta(v))} - \frac{72z^4\bar{\mu}^2}{(1+6\bar{\mu}\tilde{z}^2 v\theta(v))} + \frac{M}{uv}\right].
\end{aligned}
\tag{A.4}
$$

Inserting the expressions (2.9) with $\cosh[n(Y-\eta)] = \frac{2^{n-1}}{(\mu\tau)^n}\left(u^n\bar{\mu}^n + v^n\bar{\mu}^n\right)$ and $\mu\tau = 2\bar{\mu}\sqrt{uv}$ and reinstating appropriate $\theta$ functions for every appearance of $u, v$, we obtain power series' in $v$ for the above $f_5, f_6, f_7$, respectively. Using these, we have verified *a posteriori* that our solution obeys eq. (A.3) order by order in $v$ (and by symmetry also in $u$), and hence all $\delta$-functions in the Einstein equations cancel to that order. Since (2.8) is a solution to the Einstein equations for arbitrarily small proper times, this implies that we know the metric for $u \ll 1, v \ll 1$, including $u = 0, v = 0$.



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As a consequence of that, we are able to calculate the EMT for small (positive and negative) $x^{\pm}$ by repeating the holographic renormalization procedure of section 3 for the metric in the form (A.1) to Brinkmann coordinates. One finds

$$T_{--} = -\frac{1}{2}\partial_-^2 a_0(x^+, x^-) + \frac{x^+}{x^-}\theta(x^+)\theta(x^-) \lim_{z\to 0} \frac{L-K}{2z^4} \tag{A.5}$$

which using

$$\begin{aligned} a_0(x^+, x^-) &= -2\bar{\mu}\left(x^+\theta(x^+) + x^-\theta(x^-)\right) - \frac{5+c_1}{3}\bar{\mu}^2 4x^{+2}x^{-2}\theta(x^+)\theta(x^-) + \mathcal{O}(x^{+3}, x^{-3}) \\ \lim_{z\to 0}\frac{L-K}{2z^4} &= -\frac{4}{3}(8+c_1)\bar{\mu}^2 x^+x^-\theta(x^+)\theta(x^-) + \mathcal{O}(x^{+2}, x^{-2}) \end{aligned} \tag{A.6}$$

becomes

$$T_{--} = \bar{\mu}\delta(x^-) - 4x^{+2}\bar{\mu}^2\theta(x^+)\theta(x^-) + \mathcal{O}(x^{+3}, x^-). \tag{A.7}$$

Note that in order for holographic renormalization to be consistent, all terms proportional to $z^2$ in $L-K$ have to cancel. We have explicitly verified this up to $\mathcal{O}(\tau^{10})$. By symmetry we have

$$T_{++} = \bar{\mu}\delta(x^+) - 4x^{-2}\bar{\mu}^2\theta(x^+)\theta(x^-) + \mathcal{O}(x^{-3}, x^+), \tag{A.8}$$

and from (3.4) we can glean

$$T_{+-} = \frac{1}{2}\left(T_{\tau\tau} - \tau^{-2}T_{\eta\eta}\right) = 8x^+x^-\bar{\mu}^2\theta(x^+)\theta(x^-) + \mathcal{O}(x^{-2}, x^{+2}). \tag{A.9}$$

This can be used to calculate the energy density in the more familiar $t, z$ coordinates. The transformation is straightforward and one finds

$$\begin{aligned} T^{00}(t,z) &= \frac{1}{2}\left(T^{++} + 2T^{+-} + T^{--}\right) \\ &\simeq \frac{1}{2}\bar{\mu}\left(\delta(x^+) + \delta(x^-)\right) - 2\bar{\mu}^2\theta(x^+)\theta(x^-)\left(x^{+2} + x^{-2} - 4x^+x^-\right) \\ &\simeq \frac{1}{\sqrt{2}}\bar{\mu}\left(\delta(t+z) + \delta(t-z)\right) + 2\bar{\mu}^2\theta(t^2 - z^2)\left(t^2 - 3z^2\right) . \end{aligned} \tag{A.10}$$

Taking the result (to this order of calculation) at face value, for fixed time $t$ the energy density deposited in the forward lightcone is positive at the collision point $z = 0$ (corresponding to mid-rapidity $\eta = 0$), while it becomes negative for some $z^2 \lesssim t^2$, especially close to the (positive) $\delta$-functions at $z^2 = t^2$. We believe this is allowed for a quantum field theory, as discussed in section 4, but concede that the physical interpretation of this result demands further study. Calculating the total energy at any given time $t$ gives

$$E(t) \equiv \int_{-\infty}^{\infty} dz T^{00}(t,z) = \sqrt{2}\bar{\mu}, \tag{A.11}$$

up to the accuracy of the approximation. Therefore, the total system energy is conserved after the collision, which serves as yet another check on our result.

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